

① Dibujar, **razonadamente**, la gráfica de una función real que cumpla la(s) siguientes condiciones:

- a) $\lim_{x \rightarrow -1} f(x) = -2$ c) $\lim_{x \rightarrow 0} f(x) = \frac{1}{2}$ e) $\lim_{x \rightarrow 2^-} f(x) = +\infty$
b) $\lim_{x \rightarrow 2} f(x) = -1$ d) $\lim_{x \rightarrow 0} f(x) = -\infty$ f) $\lim_{x \rightarrow -1^-} f(x) = 0$
g) $\lim_{x \rightarrow 2} f(x) = -1$; $\lim_{x \rightarrow 0} f(x) = 0$; $\lim_{x \rightarrow -2} f(x) = 3$; $\lim_{x \rightarrow 5} f(x) = \frac{1}{2}$
h) $\lim_{x \rightarrow -\infty} f(x) = -1$; $\lim_{x \rightarrow 0^-} f(x) = 1$; $\lim_{x \rightarrow 0^+} f(x) = -\infty$; $\lim_{x \rightarrow +\infty} f(x) = -\infty$
i) $\lim_{x \rightarrow -\infty} f(x) = +\infty$; $\lim_{x \rightarrow -1} f(x) = +\infty$; $\lim_{x \rightarrow 1} f(x) = -\infty$; $\lim_{x \rightarrow +\infty} f(x) = -2$

② Calcular los límites:

- (1) $\lim_{x \rightarrow -1} x^2 - 4x - 5$ (12) $\lim_{x \rightarrow -1} \frac{\sqrt{2x^2 - 5x + 2}}{\sqrt[3]{x^3 + 9}}$
(2) $\lim_{x \rightarrow 0} \frac{2x + 3}{x - 1}$ (13) $\lim_{x \rightarrow 2} \frac{\sqrt{x^2 - 5x + 6}}{\sqrt{x^2 - 4}}$
(3) $\lim_{x \rightarrow 1} \frac{x^2 - 5x + 2}{2x^2 - 3x + 2}$ (14) $\lim_{x \rightarrow -1} \frac{x + 1}{\sqrt{x + 1}}$
(4) $\lim_{x \rightarrow 0} \frac{2x}{x}$ (15) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{\sqrt{x} - 4}$
(5) $\lim_{x \rightarrow 1} \frac{x^2 - 5x + 4}{7x^3 - 2x + 5}$ (16) $\lim_{x \rightarrow 0} \frac{1 - \sqrt{x + 1}}{x}$
(6) $\lim_{x \rightarrow 2} \frac{2x^3 - 9x^2 + 12x - 4}{x^3 - 2x^2 - 4x + 8}$ (17) $\lim_{x \rightarrow 3} \frac{\sqrt{x^2 - 4x + 12} - \sqrt{x^3 - 5x - 3}}{2x^2 - x - 15}$
(7) $\lim_{x \rightarrow -1} \frac{x^2 + 5}{x^2 - 1}$ (18) $\lim_{x \rightarrow -2} \frac{\sqrt{x^2 + 4x + 10} - \sqrt{4 - x}}{x + 2}$
(8) $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 + 3}$ (19) $\lim_{x \rightarrow 3} \frac{\sqrt{\frac{x}{3}} - 1}{x - 3}$
(9) $\lim_{x \rightarrow 1} \frac{2x^3 - 14x^2 + 12x}{x^3 - 10x^2 + 27x - 18}$ (20) $\lim_{x \rightarrow 2} \frac{\sqrt{x + 2} - 2}{\sqrt{\frac{x}{2}} - 1}$
(10) $\lim_{x \rightarrow 0} \frac{x^3 - 2x^2 + 6x}{2x^2 + 3x}$ (21) $\lim_{x \rightarrow -3} \frac{\sqrt{x^2 - 5} - 2}{x^2 + 4x + 3}$
(11) $\lim_{x \rightarrow -2} \frac{2x^2 + 7x + 6}{2x^3 + 3x^2 - 3x - 2}$ (22) • $\lim_{x \rightarrow 9} \frac{\sqrt[3]{x - 1} - 2}{x - 9}$

③ Calcular, **si existen**, los límites funcionales:

- (1) $\lim_{x \rightarrow -2} \frac{1}{x + 2}$ (6) $\lim_{x \rightarrow 5} \frac{x - 2}{(x - 5)^6}$
(2) $\lim_{x \rightarrow -1} \frac{2x^2 - 5x + 3}{x^2 + 2x + 1}$ (7) $\lim_{x \rightarrow 1} \left(\frac{1}{1 - x} - \frac{3}{1 - x^3} \right)$
(3) $\lim_{x \rightarrow 3} \frac{-2x}{x^2 - 9}$ (8) $\lim_{x \rightarrow 2} \left(\frac{x}{x^2 - 4} - \frac{1}{x^2 - 2x} \right)$
(4) $\lim_{x \rightarrow 1} \frac{x + 1}{(x - 1)^4}$ (9) $\lim_{x \rightarrow 4} \left(\frac{x + 6}{x^2 - 16} - \frac{x + 1}{x^2 - 4x} \right)$
(5) $\lim_{x \rightarrow 0} \frac{-3}{x^5}$

- (10) $\lim_{x \rightarrow 2} \left(\frac{1}{2-x} - \frac{3}{8-x^3} \right)$
- (11) $\lim_{x \rightarrow +\infty} (x^6 + 2)$
- (12) $\lim_{x \rightarrow -\infty} (x^6 + 2)$
- (13) $\lim_{x \rightarrow -\infty} (-2x^3 - 4x + 3)$
- (14) $\lim_{x \rightarrow +\infty} (-2x^3 - 4x + 3)$
- (15) $\lim_{x \rightarrow +\infty} (x^3 - 3x + 2) - (x^3 - 3x + 6)$
- (16) $\lim_{x \rightarrow -\infty} (-2x^4 - 5x^2 + 3)$
- (17) $\lim_{x \rightarrow +\infty} (-4x^6 - 5x^3 + 7)$
- (18) $\lim_{x \rightarrow +\infty} \frac{-2x^3 + 3x^2 - 7x - 2}{x^2 - 3}$
- (19) $\lim_{x \rightarrow -\infty} \frac{-2x^3 + 3x^2 - 7x - 2}{x^2 - 3}$
- (20) $\lim_{x \rightarrow +\infty} \frac{2x^6 + 2}{x^8 + 3}$
- (21) $\lim_{x \rightarrow -\infty} \frac{2x^6 + 2}{x^5 + 3}$
- (22) $\lim_{x \rightarrow -\infty} \frac{-2x^3 + 2x - 2}{3 - 3x^2}$
- (23) $\lim_{x \rightarrow -\infty} \frac{-x^4 + 2x^2 + 3x - 2}{2x^3 + 3x^4 - 7x - 2}$
- (24) $\lim_{x \rightarrow +\infty} \frac{2x^3 + 2x - 2}{3 - \sqrt{2}x^3}$
- (25) $\lim_{x \rightarrow +\infty} \frac{(2x^2 - 3) \cdot (2 - x)}{x^3 - 1}$
- (26) $\lim_{x \rightarrow +\infty} \left(x^2 - \frac{x^4 - 3x^2 + 1}{x^2 - 3} \right)$
- (27) $\lim_{x \rightarrow +\infty} (\sqrt{x+2} - \sqrt{x})$
- (28) $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 2x + 3} - x)$
- (29) $\lim_{x \rightarrow +\infty} (\sqrt{x^2 - 3x + 2} - \sqrt{x^2 + x - 1})$
- (30) $\lim_{x \rightarrow +\infty} (\sqrt{x^2 - 3x + 1} - \sqrt{x^2 + 1})$
- (31) $\lim_{x \rightarrow -\infty} (\sqrt{2-x} + x)$
- (32) $\lim_{x \rightarrow -\infty} \left(\frac{\sqrt{x^2 + 4} - 2}{\sqrt{x^2 + 9} - 3} \right)$
- (33) $\lim_{x \rightarrow -\infty} (\sqrt{x^2 + 4x + 5} - \sqrt{x^2 + 2})$
- (34) $\lim_{x \rightarrow +\infty} (\sqrt{x + \sqrt{x}} - \sqrt{x - \sqrt{x}})$
- (35) $\lim_{x \rightarrow +\infty} \frac{\sqrt[5]{32x^5 + 4x^2 - 3} + \sqrt{9x^2 - 3}}{2 - 5x}$
- (36) $\lim_{x \rightarrow +\infty} \frac{1}{\sqrt{x^2 - 5x + 3} - x}$
- (37) $\lim_{x \rightarrow +\infty} \frac{\sqrt{x-1} - x}{\sqrt{x+1} - \sqrt{4x+2}}$
- (38) $\lim_{x \rightarrow +\infty} \frac{\sqrt{x+2} - x}{\sqrt{x+1} - \sqrt{x-1}}$

④ Calcular, **si existen**, los límites funcionales:

- (1) $\lim_{x \rightarrow -\infty} 2^{-2x^2 - 7x + 1}$
- (2) $\lim_{x \rightarrow -\infty} (-5x)^{3x^3 + 2x - 1}$
- (3) $\lim_{x \rightarrow -\infty} \frac{3^x}{2^x}$
- (4) $\lim_{x \rightarrow +\infty} \frac{3^x}{3^{x+4}}$
- (5) $\lim_{x \rightarrow 3} \left(\frac{x^2 - 9}{x - 3} \right)^{\frac{x+2}{(x-3)^2}}$
- (6) $\lim_{x \rightarrow +\infty} \left(\frac{5x^2 - 6x + 1}{7x^2 + 2} \right)^{3x-1}$
- (7) $\lim_{x \rightarrow -\infty} \left(\frac{4x^3 - 6x^2}{4x^3 - 1} \right)^{\frac{x^2 + 1}{x + 2}}$
- (8) $\lim_{x \rightarrow +\infty} \left(\frac{2x^2 - 7x + 1}{2x^2 + 2} \right)^{3x^2 + 5}$
- (9) $\lim_{x \rightarrow 0} \left(\frac{x^2 - 3x + 4}{x^2 + 3x + 4} \right)^{\frac{1}{3x}}$
- (10) $\lim_{x \rightarrow 0} \left(\frac{2x^2 - 5x + 7}{2x^2 + 2} \right)^{\frac{x^2 - 1}{x^2}}$
- (11) $\lim_{x \rightarrow 2} \left(\frac{3x^2 - 5x - 2}{2x + 5} \right)^{\frac{x-4}{2}}$
- (12) $\lim_{x \rightarrow +\infty} \left(\frac{3x^2 - 5x + 1}{4x^2 - 7} \right)^{-\sqrt{x}}$
- (13) $\lim_{x \rightarrow +\infty} \left(\frac{3x^2 - 5x + 1}{3x^2 + 7} \right)^{x+3}$
- (14) $\lim_{x \rightarrow 0} \left(\frac{2x^2 - 5x + 2}{x + 2} \right)^{\frac{x+2}{x^2}}$
- (15) $\lim_{x \rightarrow +\infty} \left(\frac{2x^2 + 3x + 2}{x^2 - 2} \right)^{-x^2 + 1}$
- (16) $\lim_{x \rightarrow 1} \left(\frac{3x^2 - x + 2}{x + 2} \right)^{\frac{x^2 - 1}{x - 1}}$

⑤ Calcular las asíntotas de las funciones y representar, aproximadamente, sus gráficas.

$$a) f(x) = \frac{1}{x-4}$$

$$e) f(x) = \frac{2}{x^2+1}$$

$$h) f(x) = \frac{1}{x^3-3x^2}$$

$$b) f(x) = \frac{-1}{x-4}$$

$$f) f(x) = \frac{x}{x^2+1}$$

$$i) f(x) = \frac{7x^2-3x-4}{1-2x}$$

$$c) f(x) = \frac{3x}{x-3}$$

$$d) f(x) = \frac{x^2}{x^2+1}$$

$$g) f(x) = \frac{x^2-1}{x-1}$$

$$j) f(x) = \frac{x^4+x^2}{x^3+1}$$

$$k) f(x) = \frac{1}{(x-1)^2(x+3)(x-5)}$$

$$l) f(x) = \frac{2x^3+3x^2+3x+1}{x^3+4x^2+4x}$$